The Effect of Load VSWR on Transmission Line Loss

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The purpose of this memo is to clear up any confusion that may arise regarding the loss of a transmission line with mismatched load impedance.

For example, several sites on the internet allude to an additional transmission line loss resulting from the greater effective values of voltage and current on lines with standing waves. The American Radio Relay League Handbook has a chapter on transmission lines that states this additional loss is because of an increase in ohmic loss (I^2R) with a greater effective current and an increase in dielectric loss (E^2/R) with a greater effective voltage. They include an equation to calculate the total transmission loss (but without derivation) as follows:

$$Total\ Loss\ (dB) = 10\ log\left(\frac{a^2-|\rho|^2}{a(1-|\rho|^2)}\right) \quad (1)$$

where

$$|\rho| = \frac{SWR-1}{SWR+1}$$

$$a = 10^{ML/10} = matched - line\ loss\ ratio$$

and

ML = the matched-line loss for a particular length of transmission line, in dB

SWR = the standing-wave-ratio at the load end of the line

The additional loss caused by the standing waves is then:

$$Additional\ Loss\ (dB) = Total\ Loss\ (dB) - ML \quad (2)$$

What's the rationale for this additional loss? For radar systems, the total loss of a transmission line terminated in a mis-matched load is normally calculated as:

$$Total\ Loss = ML - 10\ log(1-|\rho|^2) = ML + 10\log\left(\frac{1}{(1-|\rho|^2)}\right) \quad (3)$$

The answer turns out to be that the equation for additional line loss attributed to standing-waves implicitly assumes there is an ATU (antenna tuning unit) at the transmission line input. A complex-conjugate impedance is introduced by the ATU to provide a match at the transmission line input at a specific frequency. Under these conditions, the portion of the transmitted signal that is reflected from the mismatched load can be viewed as arriving at the ATU where it is reflected back toward the load to be partly absorbed and partly reflected again. This will continue with ML attenuation each way until the original reflection has diminished to zero. Therefore the loss due to the impedance mismatch at the load is accounted for as an increase in transmission line loss rather than as a separate mismatch loss. Because some of the initially reflected power ends up being absorbed by the load, it's obvious that the total loss in this case is less than it would be for systems that match to the characteristic impedance of the transmission line and absorb the reflected energy. A radar front end with a 4-port circulator presents a broad-band match to the transmission line impedance and therefore (3) is the correct expression to use for that case.

To verify that the ATU scenario described above is the correct explanation, the expression for total loss in (1) will be derived based on multiple reflections with matched transmission line attenuation (ML). An input power of unity with variables α and ρ as defined in (1) will be used.

$$Total\ Load\ Power\ (numeric) = \frac{(1-|\rho|^2)}{\alpha} + \frac{|\rho|^2(1-|\rho|^2)}{\alpha^3} + \frac{|\rho|^4(1-|\rho|^2)}{\alpha^5} + \ ....$$

$$= \frac{(1-|\rho|^2)}{\alpha}\left(1 + \frac{|\rho|^2}{\alpha^2} + \frac{|\rho|^4}{\alpha^4} + \frac{|\rho|^6}{\alpha^6} + \ ...\right) \qquad (4)$$

Recognizing that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ ...$

$$Total\ Loss\ (dB) = -10\log\left(\frac{\alpha(1-|\rho|^2)}{\alpha^2-|\rho|^2}\right) = 10\log\left(\frac{\alpha^2-|\rho|^2}{\alpha(1-|\rho|^2)}\right) \qquad (5)$$

Equation (5) is the same as (1) which validates the ATU scenario. As another check note that if the ATU was placed at the load (antenna terminals), the transmission loss would be zero, α would be unity and (5) would give a loss of 0 dB due to standing waves.

The difference in loss with the ATU (neglecting its own loss contribution) as given by (1) and the loss in absorbing the reflection as given by (3) is therefore

$$Loss\ difference\ (dB) = 10\ log\left(\frac{1}{1-\frac{|\rho|^2}{\alpha^2}}\right) \qquad (6)$$

Equation (6) says what we intuitively know and that is that the greatest loss reduction from the ATU will be obtained when the transmission line loss is low and the SWR is high.