Spectrum of relict gravitational radiation and the early state of the universe

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A phenomenological model of the universe, in which, the universe was in a maximum symmetrical quantum state before the beginning of the classical Friedman expansion, is examined. The spectrum of long-wave, background, gravitational radiation is calculated in this model. The possibility of detecting this radiation in the range $10^{-3} - 10^{-5}$ Hz is promising.

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At present, the theory of quantum effects in strong gravitational fields has reached the stage of development at which it is possible to ask what was the state of the universe before the beginning of its classical expansion according to the Friedman

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law [$a(t)\propto\sqrt{t}$ for $t>0$ and $p=\epsilon/3$]; in other words, what happened before the "big bang". It is evident that only the most weakly interacting particles—gravitons produced in the pre-Friedman or early Friedman stage, which produce in the present epoch a stochastic, nonthermal, background radiation—can preserve the information about the "pre-Friedman" stage. As shown earlier,[1] if the background radiation is isotropic, the information of greatest importance and interest is contained in the exponent of the slope of the radiation spectrum

$$\alpha = \frac{d\ln n_{\mathbf{k}}(\omega)}{d\omega} = \frac{d\ln\epsilon(\omega)}{d\omega} - 3 \tag{1}$$

in the region $\nu < 10^{11}$ Hz, i.e., below the maximum in the spectrum of electromagnetic relict radiation (henceforth this region is called long-wave region). Here $n_{\mathbf{k}}(\omega)$ is the occupation number of the mode with $|\mathbf{k}| = \omega/c$, and $\epsilon(\omega)$ is the spectral energy density of the gravitational background that is integrated over the angles. For the cases examined in Ref. 1 and in this work, the background spectrum in the long-wave region is a power law ($\alpha = \text{const}$). If the background is anisotropic, then the value of α, which is defined by the first equality in Eq. (1), is a function of the angle.

The most valuable information is contained in α because it is insensitive to the physics of the transition stage at $|t|\sim t_g=\sqrt{G\hbar/c^5}$, for which we have no data. For example, a phenomenological, nonsingular model for the evolution of the universe was examined in Ref. 1, where at $t \ll -t_g$ the classical, isotropic compression obeyed the law $a(t)=a_1|t|^{q_1}$ [$a(t)$ is the scale factor in the Friedman model], at $t \gg t_g$ the classical, isotropic expansion obeyed the law $a(t)=a_2t^{q_2}$, and in the region $|t|\sim t_g$ the value of $a(t)$, because of quantum effects, passed through the minimum; the actual form of $a(t)$ in this region was not postulated. Models of this type are used in calculating the single-loop, quantum corrections for the Einstein equations.[2] Since at $|t|\sim t_g$ the space-time metric fluctuates, $a(t)$ represents the average scale factor. If the space-time curvature nowhere exceeds the Planck curvature (l_g^{-2}), we can expect that the fluctuations will not be too large.

As a result, if it is assumed that at $t=-\infty$ the gravitons were missing [or their occupation numbers $n_{\mathbf{k}}(-\infty)<1$], as $t\to+\infty$ and $\frac{1}{3}<q_i<1$, $i=1,2$, we have[1]:

$$n_{\mathbf{k}}(\omega,+\infty)=A(a\omega)^{\alpha}, \qquad \epsilon(\omega)=\frac{\omega^3}{\pi^2}n_{\mathbf{k}}(\omega),$$

$$\alpha=-2(\mu_1+\mu_2), \quad \mu_i=\frac{3q_i-1}{2(1-q_i)}, \tag{2}$$

$$A=b_1^2b_2^2\pi^{-2}2^{-\alpha}\Gamma^2(\mu_1+1)\Gamma^2(\mu_2+1)\Big[\int_{-\infty}^{\infty}dt\,a^{-3}(t)\Big]^2,$$

$$b_i=[a_i(1-q_i)^{q_i}]^{1/(1-q_i)}.$$

It can be seen that α (in contrast to A) does not depend on the unknown form of $a(t)$ at $|t|\sim t_g$. Thus, there is a basic possibility of determining q_1—the fundamental characteristic of compression at $t<0$, if α and q_2—the values pertaining to the expansion stage at $t>0$ are known [the exponent q_i is uniquely related to the equation of state for the matter $p=(\gamma_i-1)\epsilon$ by the equation $q_i^{-1}=3/2\,\gamma_i$].

Unfortunately, for the most natural the equation of state for the matter ($p=\epsilon/3$, $q_1=q_2=\frac{1}{2}$, $\alpha=-2$), the effective amplitude of the background gravitational waves $h=\sqrt{\langle h^2\rangle}$ turns out to be very small: $h\lesssim 10^{-30}$.

Let us examine another, alternative possibility for evolution of the universe in which there is a much greater number of gravitational waves in the long-wave region. We construct a model in which the universe initially was "perpetually" in the quantum state with a radius of curvature of the order of the Planck curvature, and later left this state to enter the classical expansion stage. Without postulating the form of the "total" equations of the quantum theory, including gravitation theory, we shall assume that they have a special, maximally symmetric solution for all the variables, which is different from the Minkowski space-time. In this solution $\langle g_{ik}\rangle$ describes the de Sitter space-time (constant curvature) with a certain inverse radius of curvature H that is invariant relative to the 10-parameter de Sitter group. From the point of view of quantum theory, it is more valid to use the eigenvalue for the curvature. If there are multiple solutions that have such properties (there is a group of levels H_n), we shall choose from them a solution with minimum $H=H_0$ (minimum curvature) as the most stable solution. Further, we assume that the state with $H=H_0$ is metastable. Because of fluctuations the curvature decreases, and the universe goes over to the classical, isotropic, expansion regime. Thus, the group of space-time symmetries decreases from a 10- to a 6-parameter group.

We introduce a new, fundamental, dimensionless constant $s=H_0 l_g$ and assume that $s\ll 1$ (although not too small). This assumption is necessary, since otherwise the energy density of the gravitional background would contradict the observed data (for a more precise restriction, see below). As the last hypothesis we assume that for much smaller curvatures than H_0^2 the equation of state for the matter is $p=\epsilon/3$.

Specific models of this type can be constructed by taking into account the single-loop corrections (see Ref. 2). Thus, the assumption $s\ll 1$ corresponds to a large number of "truly elementary" particles. The existence of the initial de Sitter stage was assumed earlier.[3] The uniqueness of the hypothesis given above is that it postulates a particular solution, rather than the equations, which makes the hypothesis less binding. In particular, the contribution of the single-loop corrections is generally not described by the effective equation of state, so that Gliner's initial hypothesis of $p=-\epsilon$[4] is not applicable.

As a result, we obtain the following law for evolution of the scale factor (for the plane Friedman model): $a(t)\propto\exp(H_0 ct)$ at $t\ll -t_0$ and $a(t)\propto\sqrt{t}$ at $t\gg t_0$, where t_0 determines the decay time of the initial state. We do not require the specific form of $a(t)$ in the region $|t|\sim t_0$.

We shall describe the gravitational waves by the Lifshitz equation[5] (the condition $s\ll 1$ allows us to confine ourselves to the linear approximation):

$$h_{in}=\frac{\chi_n(\eta)}{a}\exp(i\mathbf{nr})\,e_{ik}\,,\qquad d\eta=dt/a(t),$$

$$\frac{d^2 \chi_n}{d\eta^2} + \left(n^2 - \frac{1}{a}\frac{d^2 a}{d\eta^2}\right)\chi_n = 0, \tag{3}$$

where e_{ik} is the polarization tensor. This equation is not conformally invariant.[6] The solution (3) for $t \to -\infty$ ($\eta \to -\infty$), which is invariant relative to the de Sitter group, is (see, e.g., Ref. 7):

$$\chi_n = \frac{\exp(-in\eta)}{\sqrt{2n}}\left(1 - \frac{i}{n\eta}\right). \tag{4}$$

A solution such as (4) corresponds to the case of missing gravitons in the de Sitter stage. In the region $|t| \sim t_0$ the term n^2 can be dropped in Eq. (3), after which it is solved in the general form:

$$\chi_n = C_1(n)a + C_2(n)a \int d\eta\, a^{-2}(\eta). \tag{5}$$

Finally, we can see that as $t \to +\infty$ $\alpha = -4$ and

$$\epsilon(\nu) = \frac{2}{3\pi} s^2 \epsilon_0 \nu^{-1} = 1.4 \times 10^{-13} s^2 \nu^1 \ \text{erg/cm}^3\text{·Hz}, \tag{6}$$

where $\nu = \omega/2\pi$ is the frequency of the wave and ϵ_0 is the total energy density of all the massless particles (except gravitons) at the present time. In the last equation we took into account the contribution from three types of neutrinos ($\epsilon_0 = 1.68\ \epsilon_\gamma$, $T_\gamma = 2.7$ K). The effective, dimensionless amplitude of the gravitational waves is

$$h = \sqrt{\langle h^2 \rangle} \approx 3 \times 10^{-21} s \nu^{-1}, \tag{7}$$

where ν is in Hertz. The analysis shows that the gravitational waves have no effect on the helium fusion if $s \leqslant 0.3$. At $s = 0.3$ and $\nu = 10^{-4}$ Hz we have $h \approx 10^{-17}$. This is close to the level of modern experimental capabilities (see the discussion of this problem in Ref. 8).

It is remarkable that observation of the spectrum (6) would not only allow us to determine the initial state of the universe, but also to measure experimentally the s constant which is fundamental for a unified theory of all interactions.

The examined model can be generalized to the case of open and closed Friedman models. The spectrum (6) in this case does not change in the region $\nu \gg 10^{-17}$ Hz.

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