Is the Lee constant a cosmological constant?

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It is shown that the gauge theories of elementary particles with spontaneous symmetry breaking introduce into the gravitation equations a cosmological term that varies with time. The change in this term during the evolution of the universe amounts to at least 49 orders of magnitude.

As is well known, the Einstein gravitation equation

$$R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik} + g_{ik} \Lambda \qquad (1)$$

contains in the general case a term with the cosmological constant Λ. This constant is regarded as universal and independent of the state of the universe.[1,2]

The purpose of the present paper is to show that an inevitable consequence of the ideas of the presently developing unified theory of weak and electromagnetic interactions is a dependence of Λ on the temperature of the medium. For the evolving hot universe this means a dependence of Λ on the time.

1. If, as usual, T_{ik} is taken to mean the energy-momentum tensor of matter and radiation, then the quantity $g_{ik}\Lambda/\kappa$ can be treated as an energy-momentum tensor of vacuum.[2] In elementary-particle theory, however, the energy of the vacuum and its pressure (equal to its energy density taken with a minus sign) are determined only accurate to an arbitrary constant. Therefore the "old" theories of elementary particles have yielded no information whatever on the value of Λ.

The situation has changed radically after the appearance of gauge theories with spontaneous symmetry breaking, on the basis of which one can hope, with a great degree of justification, to construct the future unified theory of weak, electromagnetic, and possibly strong interactions.[3] By its very meaning, spontaneous symmetry breaking is the result of the instability of the "usual" vacuum, which becomes restructured into a new state that is energywise more favorable. The corresponding change in the energy of the vacuum depends on the temperature of the medium, decreases with increase of this energy, and vanishes at a certain critical temperature T_c.[4] By the same token, the energy of the vacuum becomes temperature-dependent, and so does consequently the cosmological constant Λ. Although this constant is defined, as before, only accurate to an arbitrary constant term, the difference $\Lambda_{(T)} - \Lambda_{(0)}$ now has a perfectly defined value. It is precisely this difference which will be discussed below.

2. To calculate the character of the dependence of Λ on the temperature T, we consider a model that is contained (with small variations) as a component part in all the models of the unified theory, and ensures the spontaneous symmetry breaking. The Lagrangian of this model is

$$L = \partial_\mu \phi^* \partial^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2, \qquad (2)$$

where ϕ is a complex scalar field. Owing to the "incorrect" sign of the term $\mu^2\phi^*\phi$, the usual solution yields at $T=0$ particles that have an imaginary mass $m_\phi = i\mu$, i.e., tachyons. This solution is unstable; to obtain a stable solution we must make the substitution $\phi = (1/\sqrt{2})(\phi_1 + i\phi_2 + \sigma)$, after which L goes over into L_σ[3]:

$$L_\sigma = \hat{L}_\sigma + \frac{\mu^2\sigma^2}{2} - \frac{\lambda\sigma^4}{4}, \qquad (3)$$

where $\hat{L}_\sigma$ is the operator part of L_σ, and $\mu^2\sigma^2/2 - (\lambda\sigma^4/4)C$ is a numerical term corresponding to the change in the energy density of the vacuum (with the sign reversed). The symmetry breaking parameter $\sigma = \sigma_{(T)}$ depends on the temperature, with $\sigma_{(0)} = \mu/\sqrt{\lambda}$,[3] and when T increases the quantity $\sigma_{(T)}$ decreases, vanishing throughout at $T > T_c$, where $T_c \sim \sigma_{(0)}$.[4] We denote the energy density of the vacuum at the temperature T by $\epsilon_{(T)}$. It follows then from (3) that

$$\epsilon_{(T)} = \epsilon_{(0)} + \frac{\lambda}{4}(\sigma^2_{(0)} - \sigma^2_{(T)})^2$$

whence

$$\Lambda_{(T)} = \Lambda_{(0)} + \frac{\lambda\kappa}{4}(\sigma^2_{(0)} - \sigma^2_{(T)})^2 .$$

3. For a numerical estimate we assume for $\sigma_{(0)}$ the same value as in Weinberg's model,[5] $\sigma_0\sqrt{\sqrt{2}/G} \sim 250$ GeV (here G is the Fermi weak-interaction constant). For the quantity λ we have an experimental bound $\lambda \gtrsim 10^{-6}$ [6]. Then at $T > T_c$ the value of $\epsilon_{(T)}$ is larger than $\epsilon_{(0)}$ by an amount $\gtrsim 10^{42}$ erg/cm^3 or, in grams, by $\gtrsim 10^{21}$ g/cm^3. At the present time, the absolute value of ϵ does not exceed 10^{-28} g/cm^3. Thus, at a sufficiently high temperature we have $\Lambda_{(T)} \gg \Lambda_{(0)}$, i.e., we can state the value of $\Lambda_{(T)}$ even though we do not know the value of $\Lambda_{(0)}$, viz., $\Lambda_{(T)} \approx \Lambda_{(T)} - \Lambda_{(0)}$. In particular, it follows from the obtained estimates that $\Lambda_{(T>T_c)} = \Lambda_{(T_c)} > 10^{-6}$ cm^{-2}, whereas at present $|\Lambda| \lesssim 10^{-55}$ cm^{-3}. This means that during the time of the evolution of the universe the cosmological constant has changed by more than 49 orders.

To be sure, almost the entire change occurs near $T_c = 10^{15}$–10^{16} deg.[4] In this region, the vacuum energy density is lower than the energy density of matter and radiation, and therefore the temperature dependence of Λ does not exert a decisive influence on the initial stage of the evolution of the universe. At the same time, the fact that Λ definitely differs from zero at a definite period of the existence of the universe makes speculations concerning a nonzero value of Λ in the present epoch more likely. In any case, it follows from the foregoing that, by discarding from the very outset the cosmological term in Einstein's equation, as was the custom even a few years ago (see also [7]), we would encounter a conflict with modern ideas of elementary particle theory.

A more complete analysis of the phase transition in gauge theories with spontaneous symmetry breaking and its consequences to cosmology will be given in subsequent papers.

In conclusion, I am deeply grateful to D. A. Kirzhnits, whose work in the study of the phase transition in gauge theories and constant interest in my work have stimulated the writing of this paper.

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Erratum: Is the Lee constant a cosmological constant? [JETP Lett. 19, 183 (1974)]

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The title of the article has been mistranslated. The correct title is "Is the Cosmological Constant a Constant?" The translation editor regrets this mistake.

