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OPTICAL CHARACTERISTICS OF ARTIFICIAL SATELLITES

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by

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FOREWORD

This report contains the results of a study conducted under the recently completed WSRN contract F05603-70-C-0014; the study, while not a specific requirement of the subject contract, was undertaken to evaluate the value in a statistical sense of those optical observations taken by WSRN teams over the past several years.

The authors wish to acknowledge the many helpful suggestions that were contributed by Dr. J. M. Zimmerman of the Science Center, North American Rockwell Corporation, Thousand Oaks, California. Dr. McCue is a Member of the Technical Staff, Science Center, North American Rockwell Corporation, Thousand Oaks, California. Dr. Williams is a Senior Engineer at the Jet Propulsion Laboratory, Calif. Institute of Technology, Pasadena, California.

ABSTRACT

This report presents a comprehensive study of the observed optical characteristics of several hundred satellites. The primary data for the study were 22,000 optical observations gathered by volunteer observers. These observations were reduced to standard observing conditions and subjected to numerous statistical tests that verified the quality of the data and identified the principal sources of variance. For instance, it was found that the observations reported by individual observers tended to be internally consistent. However, there was considerable difference in the mean magnitude reported by different observers. In general the observations tended to be of very high quality, thus indicating the ability of volunteer observers to contribute valuable scientific data. The observational results were also compared with theoretical satellite brightness predicted by equations derived by the authors. Consistent agreement with theoretical calculations was found.

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1. INTRODUCTION

In an earlier paper (Reference 1: Williams and McCue 1966) the authors discussed the analysis of satellite optical characteristics data. Data on both brightness and spin rate were considered. This paper extends the analysis of brightness information and gives particular attention to quantifying and categorizing the errors inherent in the observational data. Most of the observations used in this study are listed in two catalogs: Reference 2 which contains 11,000 observations of 365 satellites reported between 1957 and June 1965, and Reference 3 which contains 7,900 additional observations of 574 objects reported prior to May 1968. Summaries of additional data may also be found in Pilkington (References 4-7) and Meeus (References 8 and 9). Lists of the sizes and shapes of the various orbiting bodies may be found in Rees and King-Hele (Reference 10), King-Hele and Quinn (References 11, 12 and 13) and Quinn and King-Hele (Reference 14). The observations used here were made by teams of the Western Satellite Research Network (WSRN).* WSRN currently consists of

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volunteer visual observers whose efforts are directed and coordinated by North American Rockwell Corporation (Reference 15). Each team is composed of from one to ten observers, many of whom have been providing a large number of visual satellite observations since 1957. WSRN teams usually track satellites that are between fourth and tenth magnitude. Occasionally, however, sightings are at eleventh magnitude or fainter. Observations of all the Apollo Spacecraft have been made during translunar coast. Recently, six WSRN teams reported observations of Apollo 12 including several sightings at ranges in excess of 280,000 km with magnitudes between 12 and 13. Depending upon the difficulty of the object, the instruments used by WSRN observers may range in size from the naked eye to the 18-in. refractor at Granby, Massachusetts or the 20-in. reflector at Johannesburg, South Africa. Most frequently, however, observations are made with maneuverable 5-in. aperture, 20 power, wide-field refractors called apogee scopes, or with conventional reflecting instruments of 6-in, to 12-in, aperture.

A preliminary reduction of the 22,000 observations used for the study showed that the data were remarkably consistent. This was true even though the observations were collected by numerous different volunteer observers under poorly controlled conditions. It also appeared that the availability of this data afforded a unique opportunity to perform statistical tests which would allow one to make definitive statements as to the quality of the data, the possible sources of variance, and the extent of human errors and/or observer bias. This investigation is now complete and the results of the analysis yielded a rather comprehensive picture of the validity and variability one may expect when utilizing the observations of volunteer observers. It furthermore demonstrated that there was good agreement between the theoretical and the observed appearance of satellites.

2. SATELLITE BRIGHTNESS THEORY

It is a fundamental fact that the apparent brightness of an object depends upon the inverse square of its slant range R so that the apparent visual magnitude m can be represented by

$$m = -26.58 -2.5 \log [A \gamma F(\phi)/R^2] \qquad (1)$$

where A is the cross-sectional area, Y is the reflectivity, and $F(\phi)$ is a function depending upon the phase angle ϕ. The phase angle is defined as the observer-satellite-sun angle and has a value of 0° when the satellite is at full phase. $F(\phi)$ may also depend upon the orientation of the satellite. The satellite has been assumed gray and a zenithal value of 0.20 magnitude of atmospheric absorption has been adopted.

Table 1 gives the phase functions for three common shapes: the sphere, cylinder, and flat plate. For each of the three geometries both specular and diffuse reflection are considered. For diffuse reflection Lambert's law was used. In the case of the cylinder the cross-sectional area in equation (1) is that of the cylinder seen broadside on, the length times the diameter. For the specular cylinder a(t) describes the fact that during the flash of length t_o one is seeing the integrated light across a chord of the sun's disk, which was assumed to be of uniform brightness.

The cylinder and the plate have phase functions which depend upon the orientation of their axis of symmetry with respect to the sun and the observer. In spherical coordinates with the polar axis parallel to the cylinder axis or perpendicular to the surface of the plate ϕ_1 is the latitude of the sun, ϕ_2 the latitude of the observer, and ϕ is the difference in the longitudes

Shape	Reflection	$F(\phi)$	Remarks	Reference
Sphere	Specular	$\frac{1}{4\pi}$		(16) Tousey
	Diffuse	$\frac{2}{3\pi^2}\left[(\pi-\Phi)\cos\Phi+\sin\Phi\right]$		(17) Russell (16) Tousey
Cylinder	Specular	$\frac{\cos(\theta/2)\quad\alpha(t)}{4\ \Delta}$	Δ is width of reflected fan of light (≥ 0.0093 radian) $\lvert\phi_1+\phi_2\rvert \leq \Delta/2$ for flash $\alpha(t)=\begin{cases}\frac{8}{\pi}\sqrt{\frac{t}{t_0}\left(1-\frac{t}{t_0}\right)} & \text{during flash } (0\leq t\leq t_0)\\ 0 & \text{otherwise } (t>t_0)\end{cases}$	
		$\frac{\alpha(t)}{4\Delta}$	Maximum Flash	
		$\frac{\cos(\Phi/2)\ \alpha(t)}{4\Delta}$	Minimum Flash	
	Diffuse	$\frac{\cos\phi_1\cos\phi_2}{4\pi}\left[(\pi-\theta)\cos\theta+\sin\theta\right]$		(18) Giese
		$\frac{\cos^2(\Phi/2)}{4}$	Maximum	
		$\frac{1}{4\pi}\left[(\pi-\Phi)\cos\Phi+\sin\Phi\right]$	Intermediate Value	
		0	Minimum	

Table 1. Phase Functions for Specular and Diffuse Spheres, Cylinders, and Flat Plates

Shape	Reflection	F (Φ)	Remarks	Reference
Flat Plate	Specular	$\frac{4 \cos (\Phi/2)}{\pi \Delta^2}$	During flash ($\lvert\phi_1-\phi_2\rvert \leq \Delta/2, \theta = \pi/2$) Δ is width of reflected beam ($\geq$ 0.0093 radians)	(19) Liemohn
		0	No Flash	
	Diffuse	$\frac{1}{\pi} \sin \phi_1 \quad \sin \phi_2$	ϕ_1 and ϕ_2 same signs	(18) Giese
		0	ϕ_1 and ϕ_2 different signs	
		$\frac{1}{\pi} \cos^2 (\Phi/2)$	Maximum	

Table 1. Phase Functions for Specular and Diffuse Spheres, Cylinders, and Flat Plates (Continued)

of the two. These three coordinates are related to the phase angle Φ by means of

$$\cos \Phi = \sin \Phi_1 \sin \Phi_2 + \cos \Phi_1 \cos \Phi_2 \cos \theta \tag{2}$$

In the cases where the phase function is dependent upon the orientation of the object the extrema of $F(\Phi)$, subject to the constraint (2), are presented in Table 1. Because the magnitude of an object increases as it becomes fainter, a maximum of $F(\Phi)$ which is a maximum of the brightness corresponds to the minimum magnitude for a given t while a minimum of $F(\Phi)$ and brightness is the same as the maximum magnitude. For a diffuse cylinder tumbling end over end about a fixed spin axis the maximum $F(\Phi)$ which is observed during a tumble will lie between the maximum $F(\Phi)$ for all possible orientations of the cylinder given in Table 1 and the intermediate value of $F(\Phi)$ (Reference 20). Similarly the minimum observed $F(\Phi)$ during a tumble lies between the intermediate $F(\Phi)$ and the minimum $F(\Phi)$ in the table. This result is due to the fact that a tumbling cylinder doesn't take on all possible orientations since one degree of freedom is constrained by the fixed spin axis.

The magnitudes of the various geometries of Table 1 have been plotted in Figure 1 for a slant range of 1000 statute miles. Magnitudes at this range will henceforth be referred to as absolute magnitudes. For the sphere, diffuse plate, and diffuse cylinder γA was taken as 1 m^2. For a minimum A of 0.0093 radian, which is the diameter of the sun, the absolute value of the specular plate would lie 11.92 magnitudes brighter than the plotted minimum flash for the specular cylinder. The curves plotted for the specular cylinder used γA $\alpha / \Delta = 1$ m^2 but if γA were taken as 1 m^2, A = 0.0093 radian, and a at its maximum $4/n$ they would be 5.34 magnitudes brighter. It is clear that

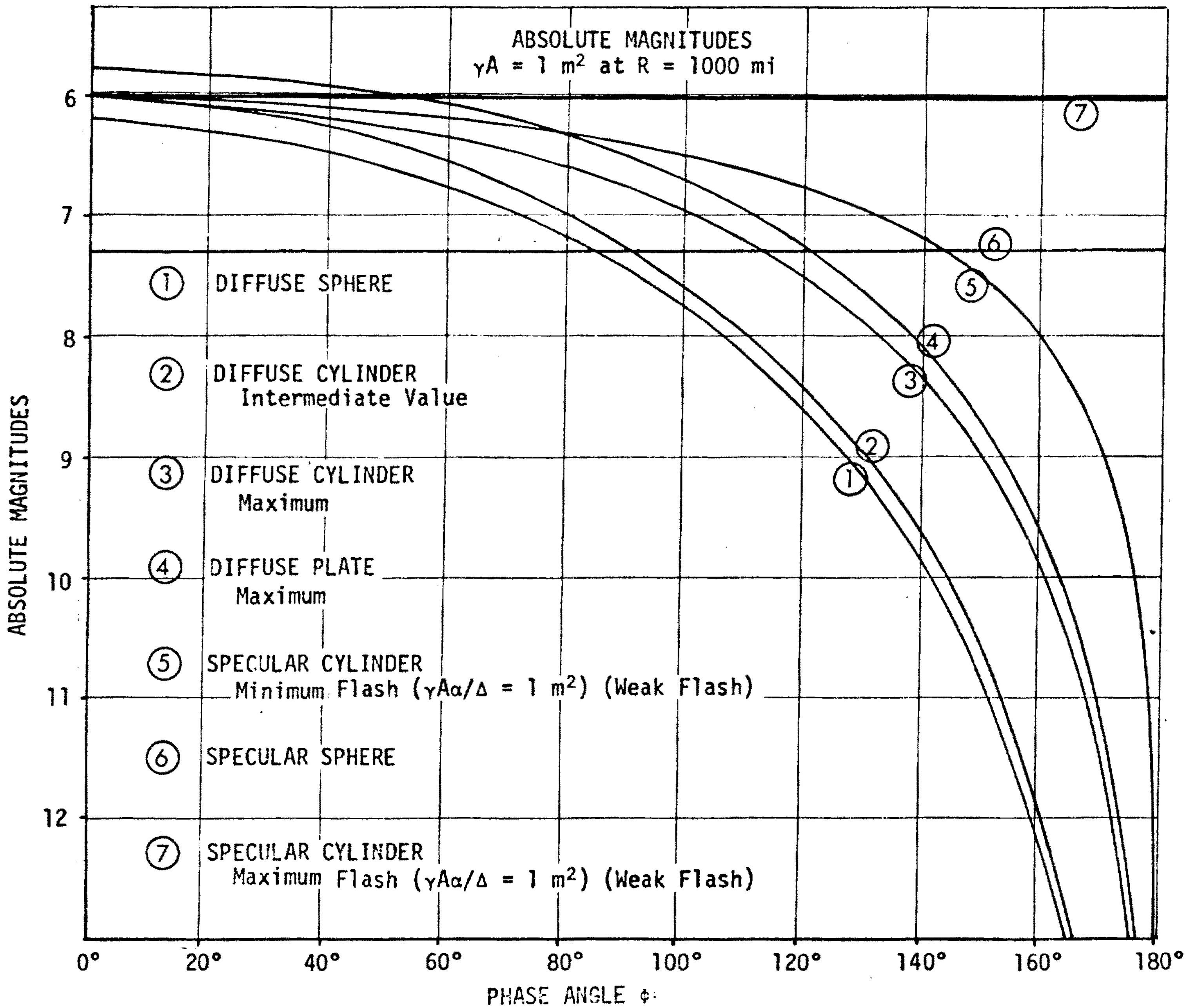

FIGURE 1. Theoretical Determination of Absolute Magnitude as a Function of Phase Angle

specular plates and cylinders with small areas or small reflectivities can give off very bright flashes. It will be shown in a later section that such phenomena are observed, particularly from spacecraft with solar cells.

3. STATISTICAL PROPERTIES OF THE OBSERVATIONAL DATA

Before comparing the observational data with the above theoretical calculations the authors conducted a detailed inquiry into its quality and possible sources of variance. As noted above, the primary data for the study were estimates of the brightness of artificial satellites that were contributed by numerous volunteer observers located throughout the world. Each observer estimates the maximum and minimum brightness (on a magnitude scale) based upon visual observations by naked eye, through binoculars, or utilizing numerous kinds of telescopes. It is clear that few aspects of the data collection are controlled. One would therefore expect to find numerous sources of error; e.g., observer bias, weather conditions, type of instrument used, qeographical location, and sky brightness. A first concern, therefore, was to conduct a preliminary reduction of all observations in order to determine the advisibility of further work.

For each of 292 satellites having more than 10 observations the data in the catalogs (References 2 and 3) were plotted several ways in order to permit visual inspection, and were then used as the basis of several tests. The decision to eliminate all objects with a small number of observations was arbitrary and may have led to biases in our sample. In particular, it probably eliminated most vehicles with a short orbital lifetime from our sample. Since these were mostly Russian Cosmos and U.S. Agena satellites this procedure introduced bias. However, it was certain that the sample was already biased -- the brighter and more interesting objects being dominant.

Prior to plotting, each of the 22,000 observations was reduced to stand observing conditions. This allowed computation of an absolute magnitude at

1000 statute miles slant-range and solar phase angle equal to 90 degrees. The standardized observations of satellite #49 (the Echo 1 balloon) are plotted in Figures 2 through 5. Since certain kinds of satellites exhibit phase angle dependence of brightness, the maximum (+) and minimum (o) magnitudes were plotted as a function of phase angle (Figures 2 and 3). For each plot a linear regression line was computed to see if any phase angle dependence of brightness existed. The standard deviation of the observations about the regression line was also computed.

The small positive slopes indicated in Figures 2 and 3 were not found to be significant at the 0.05 confidence level. However, the existence of such slopes would be consistent with the findings of Emmons, Rogers, and Preski (Reference 21) that the reflectivity of Echo 1 is 96% specular and 4% diffuse, The typically 1 magnitude variation between maximum and minimum magnitude must represent a variation of the specular component. Since the ratio of the diffuse to specular components is higher for the maximum magnitude the slope of Figure 2 would be expected to be larger as is the case. Figures 4 and 5 plot the maximum and minimum magnitude versus date. The objective here was to see if a time trend indicating a change in the physical characteristics of the satellite was in evidence. Over the 3,000 day time interval shown in these figures this particular satellite remained remarkably steady. Note that both regression lines are nearly horizontal.

Each of the regression lines computed as shown in Figures 2 through 5 was utilized as a basis for computing the population variance σy_x under the assumption of homoskedasticity. It was usually found that the standard deviations differed for the maximum and minimum magnitude estimates, and

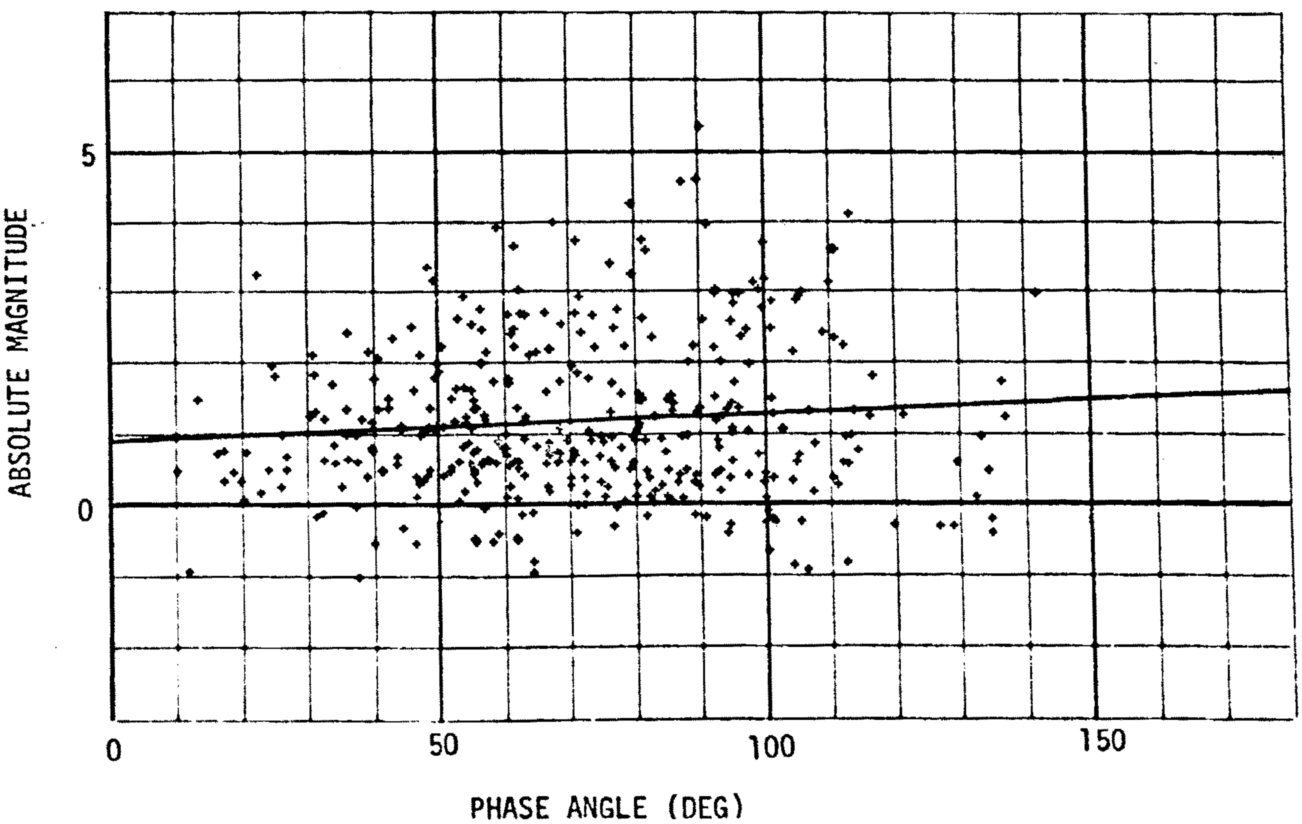

Figure 2. Maximum Absolute Magnitude vs Phase Angle for Satellite #49 (σ_{xy} = 1.11 Mag.) 383 Observations

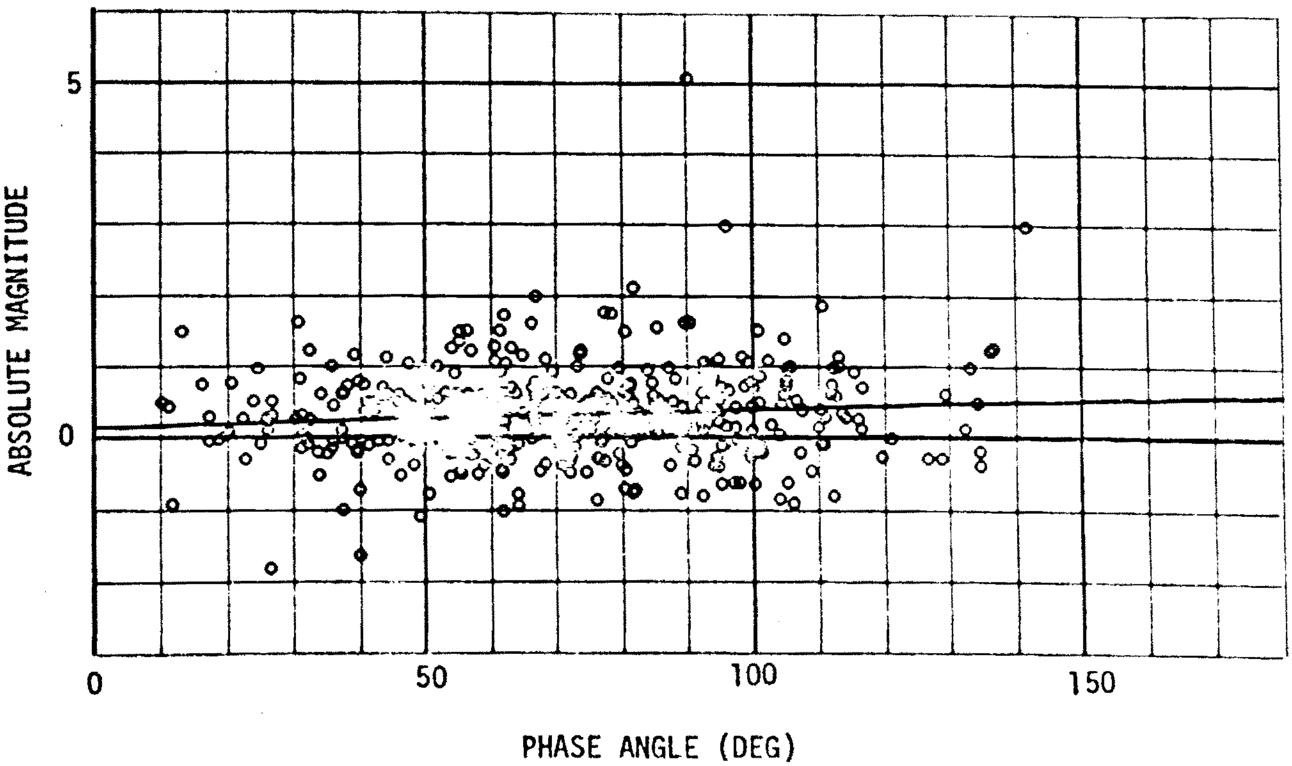

Figure 3. Minimum Absolute Magnitude vs Phase Angle for Satellite #49 (σ_{xy} = 0.65 Mag.) 404 Observations

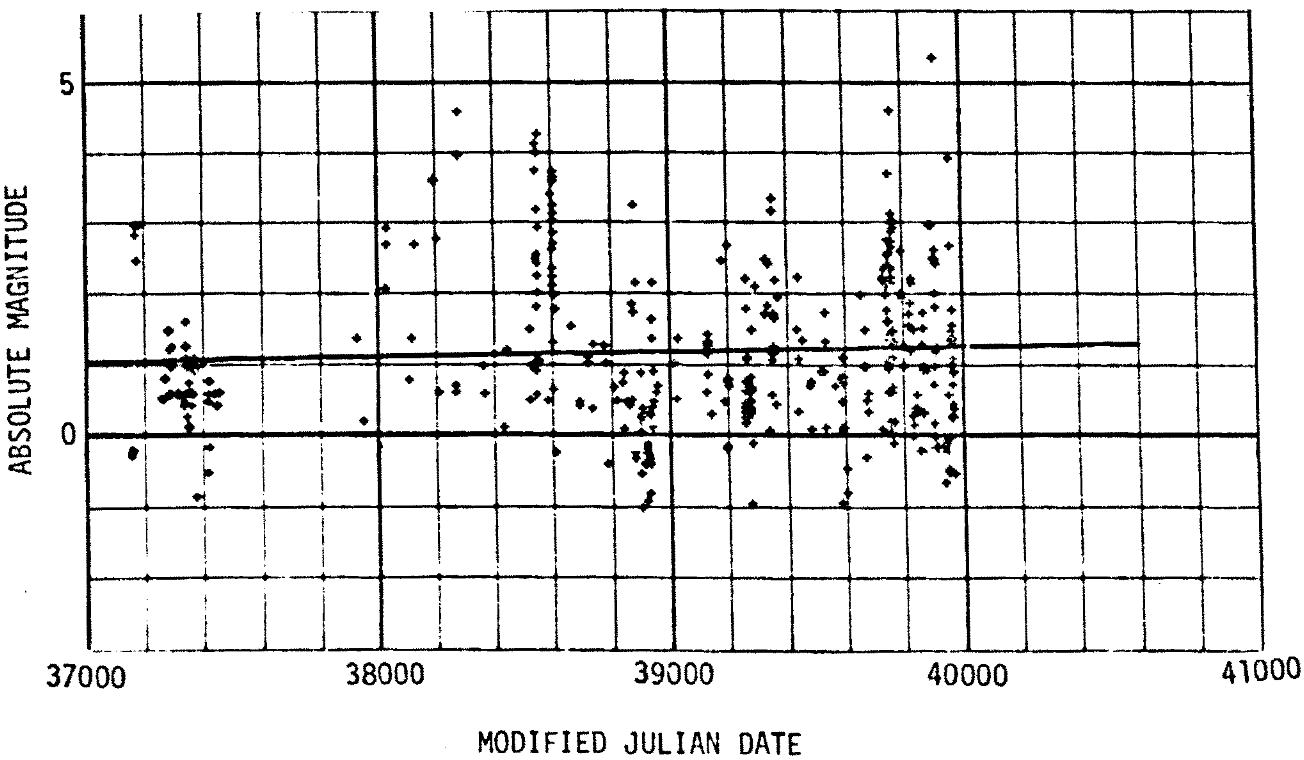

Figure 4. Maximum Absolute Magnitude vs Date for Satellite #49 (σ_{xy} = 1.10 Mag.) 387 Observations

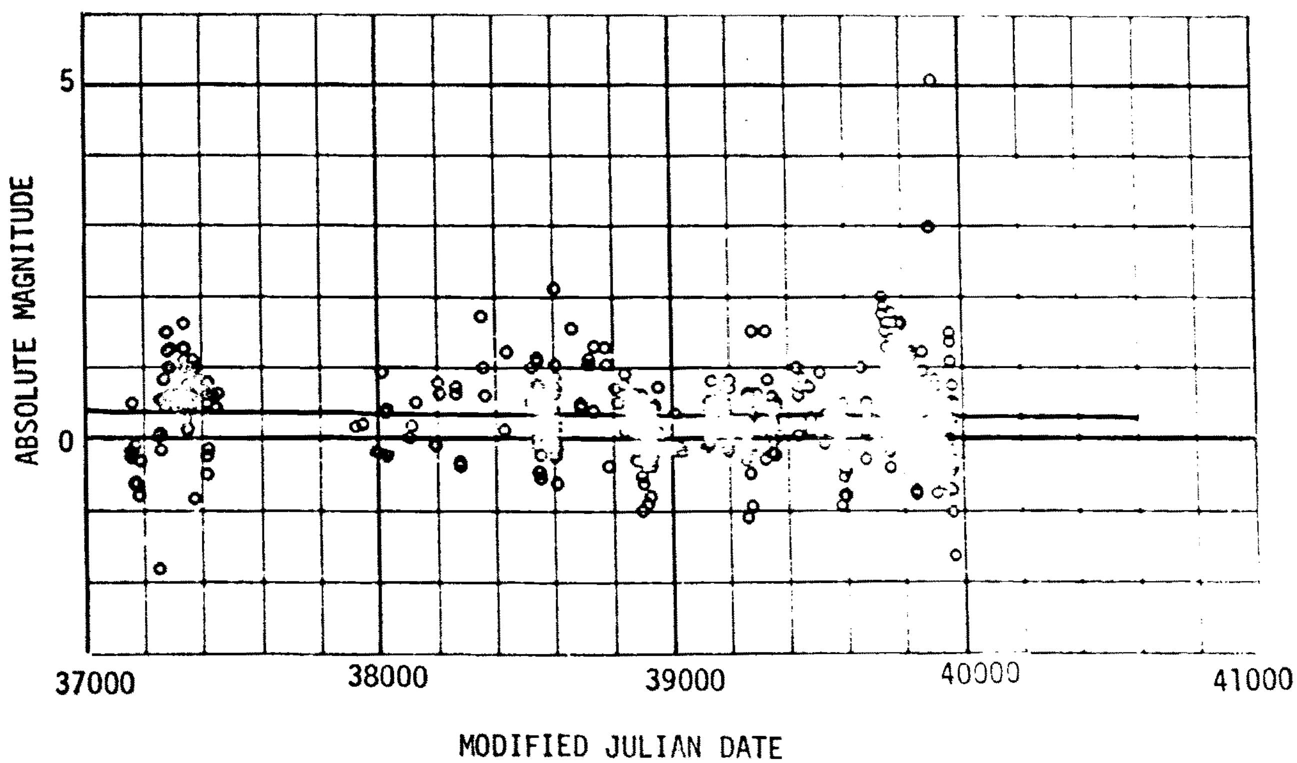

Figure 5. Minimum Absolute Magnitude vs Date for Satellite #49 (σ_{xy} = 0.66 Mag.) 408 Observations

that the values of σ_{xy} were sufficiently small to make the data useful. This same kind of consistency was repeated for most of the 292 satellites examined. This was indeed encouraging for it Indicated that the basic data were of sufficient quality to warrant more detailed examination of the sources of variance.

The distribution of satellite magnitudes and the variance of the observations of each were the subject of considerable study. These data are presented graphically in Figures 6a and 6b wherein the mean absolute magnitudes are plotted as a function of satellite number. In these figures a circle identifies the mean absolute mannitude and the vertical line with bars is used to denote the extent of the ± one standard deviation limits.

One quickly notices that the data tend to fall within an absolute magnitude band between 4 and 10. The exceptions are 3 large aluminized balloons and several large rocket bodies. The data of Figures 6a and 6b also lend themselves to a test concerning the performance of satellite observers. One may postulate that observers will make errors of different size in estimating satellite brightness and that these errors are a function of the brightness of the object observed.* That is, we might expect an observer of the Echo balloon satellite to make the largest errors because he usually reports naked eye observations or, on the other hand, we might hypothesize that the observer using a telescope and observing vary faint satellites will make the largest observational errors when estimating brightness. In Figure 7 the data of Figures 6a and 6b have been replotted to

*Here we are assuming that large absolute magnitudes imply large observed magnitudes. The two magnitudes are correlated but they are not the same; e.g., the Molniyas have bright absolute magnitudes but faint observed magnitudes. Here we are testing standard deviation vs absolute magnitude.

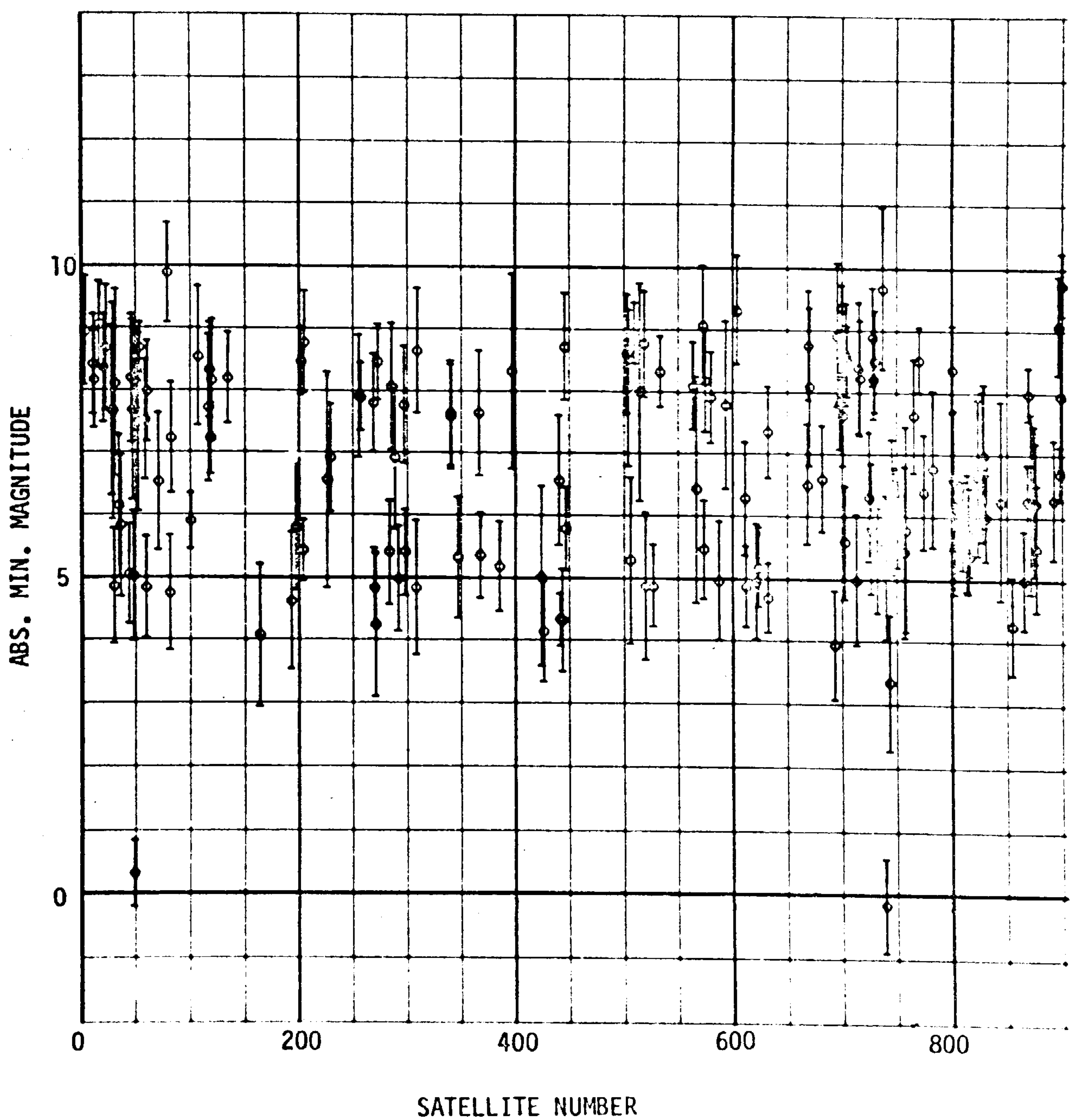

Figure 6a. Absolute Magnitude Versus Satellite Number for 292 Satellites in Catalog (±1 Standard Deviation Limits Noted)

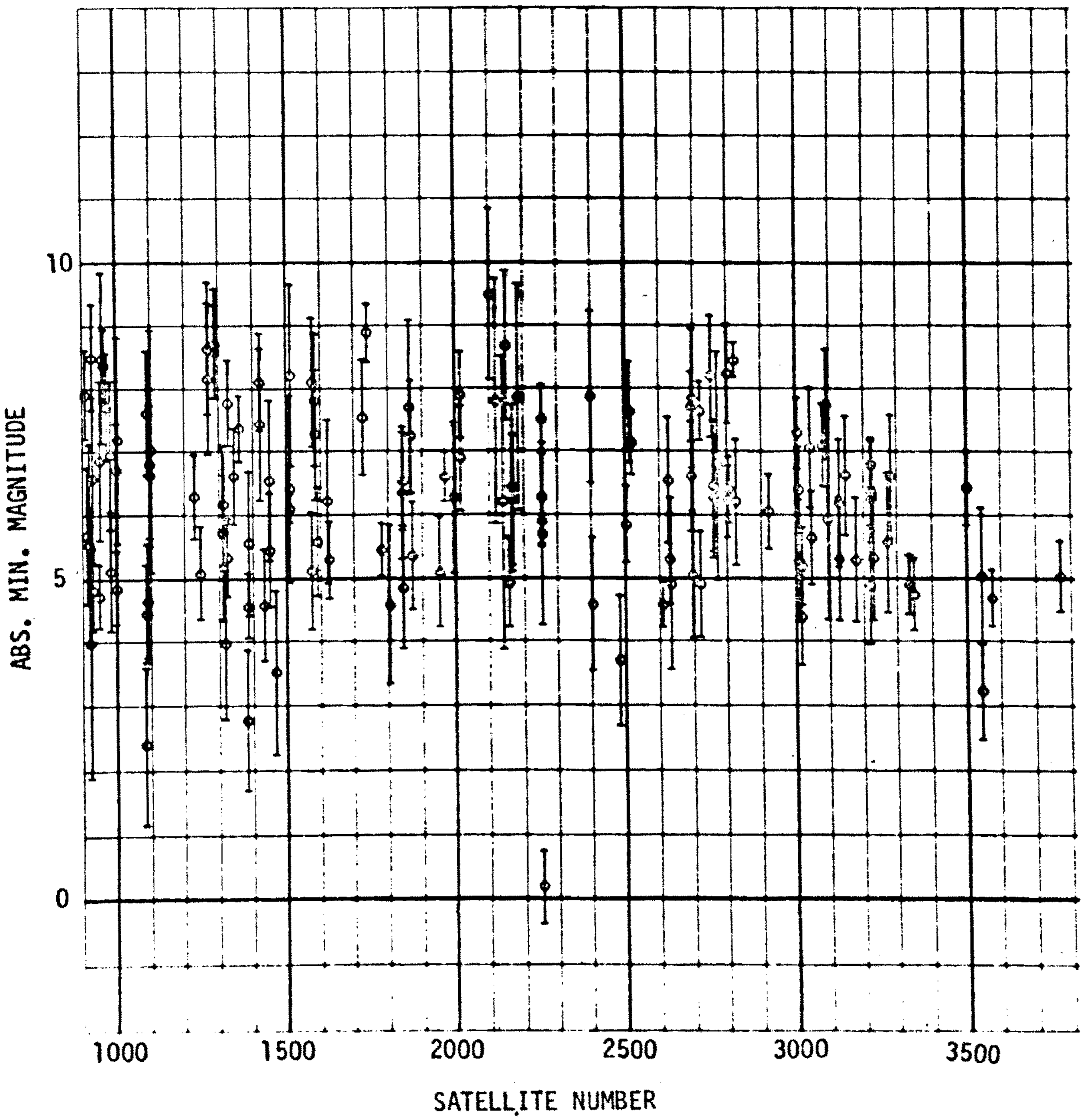

Figure 6b. Absolute Magnitude Versus Satellite Number for 292 Satellites in Catalog (±1 Standard Deviation Limits Noted)

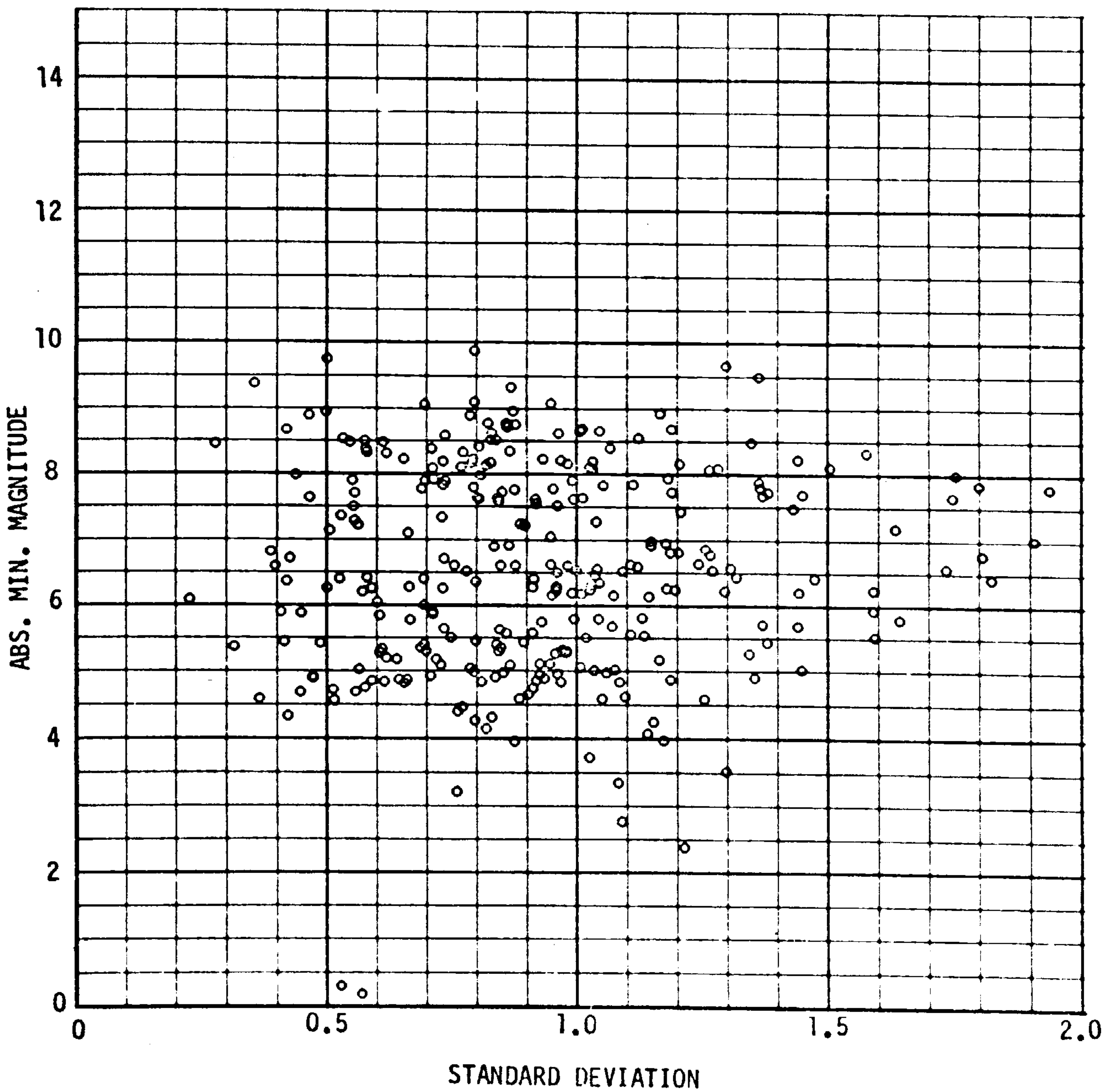

Figure 7. Scatter Diagram of Absolute Magnitude vs Standard Deviation for 292 Satellites in Catalog

show mean absolute magnitude as a function of the standard deviation of absolute magnitudes. A quick glance at this figure reveals no apparent trend that would substantiate the hypothesis above.

To quantitatively test the hypothesis we employed analysis of variance. The data were divided into ten equal groups encompassing various magnitude bands and analysis of variance was attempted using the computer program BMD07D (Reference 22). The procedure was to hypothesize that each group of observations had the same mean standard deviation. A significance level of 0.05 was chosen and an F statistic which was the ratio of the means-squared-for-means to means-squared-for-within-groups was computed under the assumption of random selection from normal populations having equal variance. Assuming the above hypothesis to be true, the F distribution is as given in Table A7 of Reference 23. The computed F statistic was considerably less than the allowable value and therefore the hypothesis of equal means was accepted. This result is important for it means that observational errors committed by the volunteer observers are independent of the absolute magnitude of the satellite observed. It therefore removes a possible source of systematic bias from consideration. It also leads us to give more credibility to the observations since we would hope that well trained observers would perform as the tests indicated they did.

With 20,000 observations it was decided not to attempt a complete reduction on all objects. Therefore, after performing the preliminary work noted above, the procedure employed was to choose certain satellites for analysis of certain questions. It was possible to perform several tests and to reach several definitive conclusions. These results appear below.

One set of analyses that is quite relevant to the tests reported above concerns the normality of the residuals. If we postulate that the observed errors are purely observational errors or small random discrepancies due to random satellite orientation we might expect the residuals to be normally distributed. To test this hypothesis several chi squared tests of normality were performed. The bright balloon satellites (Echo 1, Echo 2, and Pageos) were chosen because they should exhibit very little brightness change with phase angle. When the observations for the three satellites were tested, a chi squared test at the 0.01 significance level indicated non-normality of residuals for all three satellites. There was a strong suspicion, however, that this result could be in part due to observer and station bias. For this reason the observations of one individual observer were chosen for further testing. These new tests indicated normality at the 0.01 significance level.

One of the most interesting features of the satellite observations is that they are made by numerous different stations and observers. One would therefore be interested in knowing if different observers exhibit systematic bias in the reporting of observations. To test this proposition the observations of the Echo 1 balloon were sorted into groups according to station number. The 10 stations reporting the greatest number of satellite observations were then utilized for analysis of variance. A total of 420 observations by the 10 stations were processed through program BMD07D. This time the hypothesis to be tested is that the observations from each station exhibit equal mean absolute magnitudes. Since each station is observing the same object, and since the observations are made over a great number of geometries and a large spread in time, one would expect them to have equal mean brightness. Therefore, if the means are significantly different we may conclude that station bias is present within the data. Inspection of the maximum magnitude data indicated a noticeable difference in the observation reporting patterns of the 10 stations. Analysis of variance yielded an F ratio of 12.6 which was much larger than $F_{.95}$ (9,405) = 1.88 appearing in Table A7 of Reference 23, We, therefore, reject the hypothesis of equal station means. Practically speaking, this means that the different observers exhibit statistically significant bias in the reporting of their observations. This is an important result which must be incorporated into future studies of satellite optical characteristics. The above test also held true for the minimum magnitude data. An F of 21.6 again caused us to reject our hypothesis and to admit the presence of considerable station and observer bias. Repetition of the same kind of tests for the Echo 2 Balloon (740) and Pageos 1 (2253) again showed significant evidence of station bias when reporting magnitude.

The above three objects were all brinht naked eye satellites. To test for observer bias when observing telescopic satellites the observations of the four identical rigidized 3.65 meter balloons (81, 714, 931 and 3337) were pooled, sorted by stations, and subjected to analysis of variance tests. The results indicated the presence of highly significant observer bias when making and reporting observations of telescopic satellites.

Table 2 summarizes a final sequence of statistical tests aimed at determining aggregate observer discrimination ability when observing two or more objects under a wide variety of conditions. If we show our observers the four identical 3.65 meter balloons discussed above (81, 714, 931, 3337), the aggregate reported optical characteristics should be identical. When we perform an analysis of variance test we find that the mean absolute magnitudes

Table 2. Summary of Analysis of Variance Results
(All reported observations)

Satellite	International Designation	SPADATS Number	Absolute Magnitude	Standard Deviation	Computed F	Theoretical $F_{.95}$
Explorer 9	61-δ1	81	4.562	0.954	0.692	2.60
Explorer 19	63-53A	714	4.625	1.109		
Explorer 24	64-76A	931	4.737	0.866		
Explorer 39	68-66A	3337	4.739	0.507		
Telstar 1 rkt	62-αε2	341	7.556	1.091	2.452	2.68
Relay 1 rkt	62-βυ2	515	7.515	2.790		
Telstar 2 rkt	63-13B	575	7.880	0.893		
Relay 2 rkt	64-3B	738	8.491	1.364		
Pegasus 1	65-9A	1085	2.478	1.223	2.405	3.90
Pegasus 2	65-39A	1381	2.776	1.365		
Agena	63-3A	527	4.876	0.814	0.282	3.89
Agena	63-27A	613	4.811	0.933		
Agena	64-2A	733	5.296	0.853	2.006	4.17
Agena	64-31C	815	5.815	1.081		
Nimbus 1 rkt	64-52B	878	5.526	1.035	0.582	3.95
OGO 2 rkt	65-81B	1625	5.298	0.606		
Echo 1	60-ι1	49	0.334	0.659	68.134	3.00
Echo 2	64-4A	740	-0.231	0.881		
Pageos 1	66-56A	2253	0.193	0.650		
Echo 1	60-ι1	49	0.334	0.659	9.651	3.84
Pageos 1	66-56A	2253	0.193	0.650		
Agena	66-77A	2403	4.555	1.070	16.692	3.90
Agena	66-89A	2481	3.811	1.028		
Agena	61-σ1	163	3.929	1.423	7.135	3.86
Agena	62-κ1	271	4.271	1.344		
Agena	65-16J	1245	5.132	0.928	35.373	3.90
Pageos 1 rkt	66-56B	2255	5.838	0.560		
Agena	66-110B	2609	4.658	0.648		

reported for the four objects showed no evidence of being different at the 0.05 significance level. In other words, when our observers report sightings of these four identical objects they report the same mean absolute magnitude - - a comforting finding.

The same kind of test was performed upon a number of groups of satellites known to contain identical and different objects (see Table 2). The objects which appear in Table 2 span a wide range of magnitudes and orbit types.
The groups that were tested are noted and the computed and theoretical F ratios are also tabulated. A computed F ratio larger than the theoretical value is sufficient evidence for rejecting the hypothesis that the objects have equal mean absolute magnitudes.

In the first six cases of Table 2 the objects were believed to be identical and no evidence of magnitude difference was detected when the reduced observations were subjected to analysis of variance tests. On the other hand, two of the last five groups contained physically different members (49, 740, 2253 and 1245, 2255, 2609), two contained objects which flash (163, 271 and 2403, 2481), and one contained deformable balloons (49, 2253). In each of these cases the computed F ratio exceeded the theoretical value leading us to conclude that statistical evidence of difference in absolute magnitudes was present.

Note that even though the Fcho 1, Echo 2 and Paoeos 1 satellites have similar mean magnitudes the small difference is very significant. The 0.56 magnitude difference is about what one would expect from the difference in size and reflectivity between Echo 1 and Echo 2. Note also that the small difference between Echo 1 and Pageos 1 is significant and may be indicative of the deformations that were observed to occur in both objects or it may be indicative of observer bias. The magnitudes for these balloons are in fair agreement with the photoelectric results of Emmons, Rogers, and Preski (Reference 21), Adding atmospheric absorption to their extra-atmospheric results gives absolute magnitudes of 0.21 for Echo 1 and 0.20 for Pageos. The flashing objects in Table 2 are only observed to flash part of the time. Since there is an observational bias in the estimated magnitude of a rapidly flashing object it is not surprising that these objects show significant differences in their mean absolute magnitudes.

4. OBSERVATIONS VERSUS. THEORY

The results of the last section provided quantitative evidence of the quality of WSRN observations. This section is intended to give some examples of the appearance of several different types of satellites, and to provide some comparisons between observations and the previously developed theory. The absolute magnitude of a perfectly specular sphere is expected to have no phase angle dependence. The maximum and minimum absolute magnitudes of Echo I were presented in Figures 2 and 3. The figures are in good agreement with the expected behavior. It has already been mentioned that the small phase angle dependence which was observed is in agreement with a previously described diffuse component (Reference 21). The difference between the maximum and minimum magnitudes comes from the deformation of the balloon from a perfect sphere. Explorers 9, 19, 24, and 39 are rigidized balloons. They show no difference between maximum and minimum absolute magnitudes but they, like Echo I, were found to show a slight phase angle dependence, approximately 0.007 magnitude/deg.

For nonspherical satellites, the absolute magnitude is a function of both the phase angle and the orientation of the body. A plot of the absolute magnitude against the Phase angle still gives useful information although the range of possible values will show up as scatter in the observed points. For example, in Figure 1 the minimum absolute magnitude of a 1 m^2 diffuse cylinder would be expected to lie between the second and third curves.

If one calculates the expected phase angle dependence of the minimum absolute magnitude for a diffuse cylinder one finds that the slope must lie between 0.019 and 0.030 magnitude/deg at 90^0 phase. Figure 8 shows the observed dependence of satellite number 385, an Agena rocket. The observed

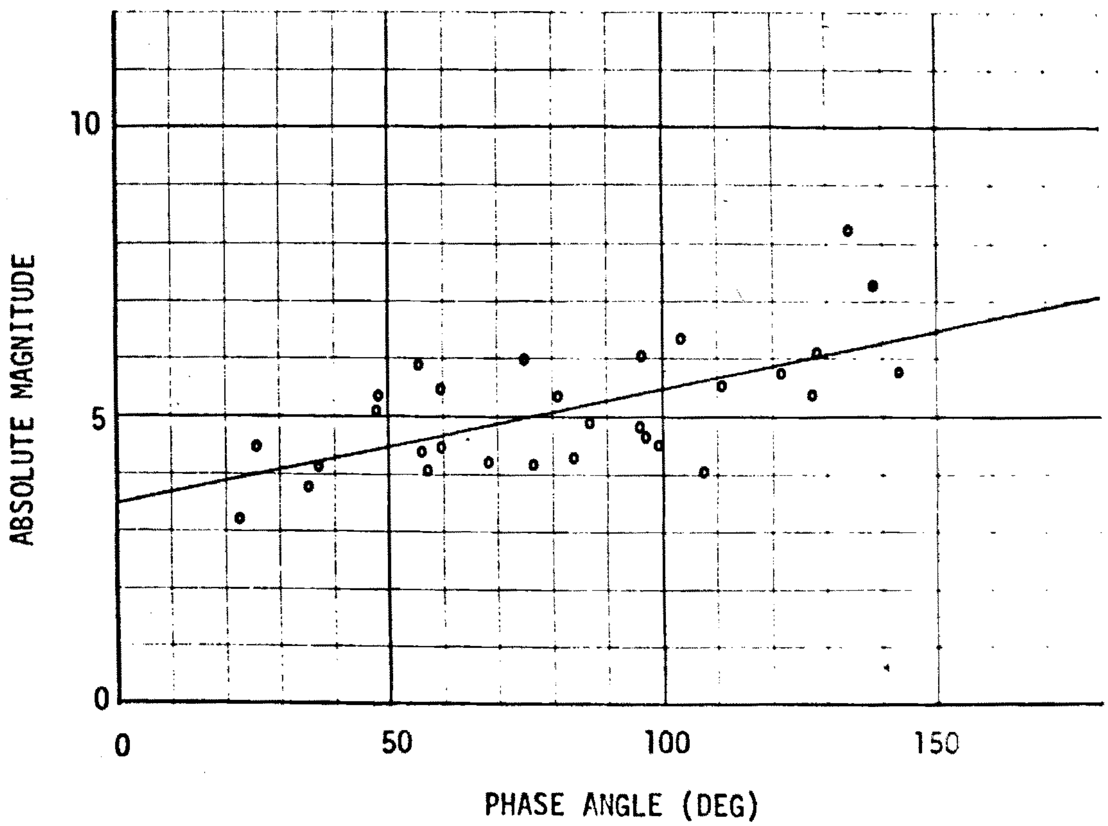

Figure 8. Minimum Absolute Magnitude Versus Phase Angle for Satellite #385 (σ_{yx} = 0.81 Mag.)
30 Observations

slope of 0.017 magnitude/deg over a large range of phases is somewhat shallower than that expected at 90^0 but the slope is close to the expected lower limit. It turns out that among the numerous well observed cylindrical rockets this object shows the steepest observed slope of the minimum absolute magnitude. Figure 9 shows the observations on a more typical object, number 613, which is also an Agena. The small slope of 0.006 magnitudes/deg is typical of the majority of cylindrical rockets though the observed slopes vary from object to object.

The minimum absolute magnitudes for most cylindrical rockets behave like specular cylinders. Because a specular cylinder concentrates its light into a narrow fan, its minimum magnitude can be very much brighter than the minimum magnitude of a diffuse cylinder of the same size and reflectivity. If an object is considered to have the reflection properties of both specular and diffuse cylinders, the specular reflectivity can be as low as 1% of the diffuse reflectivity and still dominate the minimum magnitude. Specifically, if γD is the diffuse reflectivity and γ_S is the specular reflectivity, $\gamma_s > (\pi/4)\ \Delta\gamma D$ guarantees that the specular component dominates the minimum magnitude for all phase angles. The theoretical slope for specular cylinders lies between 0.0 and 0.010 magnitudes/deg at 90^0 phase.

The observed maximum magnitudes for cylinders show a larger scatter than the minimum magnitudes. An example is shown in Figure 10 for object number 613, Since an endless cylinder can have $F(t) = 0$ such a large scatter is not unreasonable. In fact, there is observational selection against the faintest magnitudes. Of course rockets have ends and these end effects show up principally in the maximum magnitudes. Frequently the difference between the two ends is apparent to the observer, causing the maximum magnitudes to

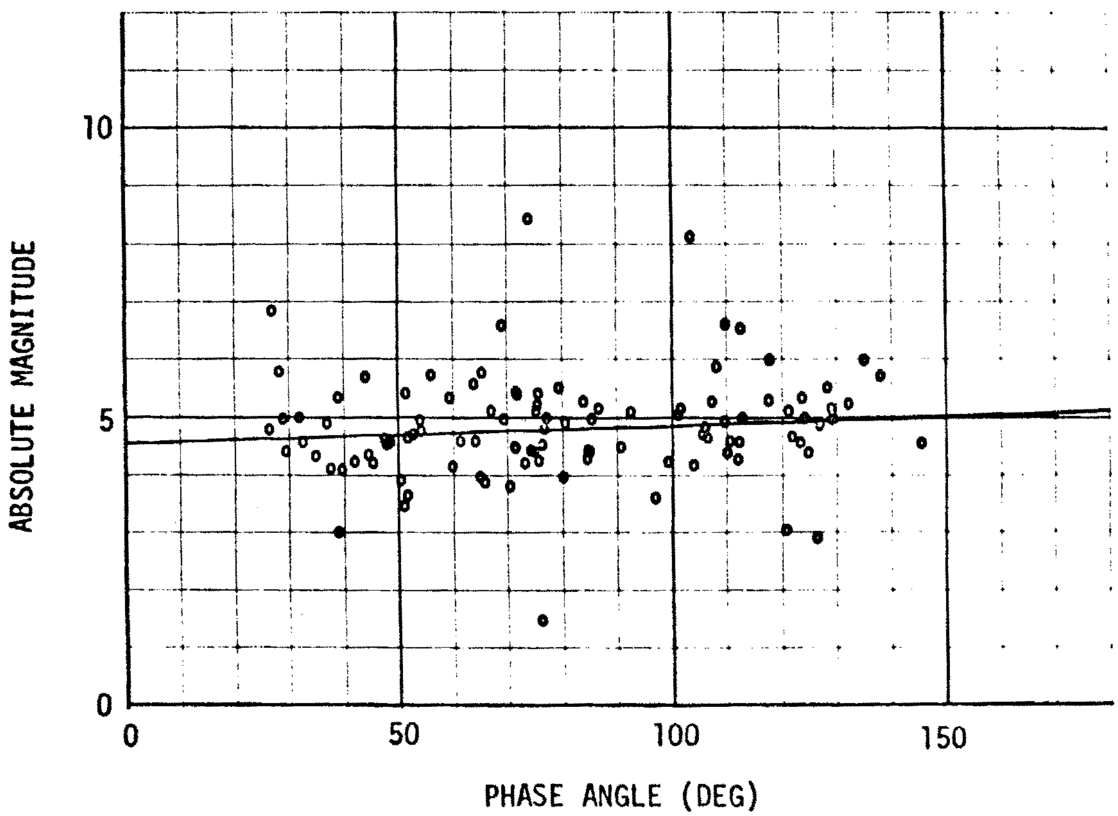

Figure 9. Minimum Absolute Magnitude Versus Phase Angle for Satellite #613 (σ_{yx} = 0.93 Mag.)
109 Observations

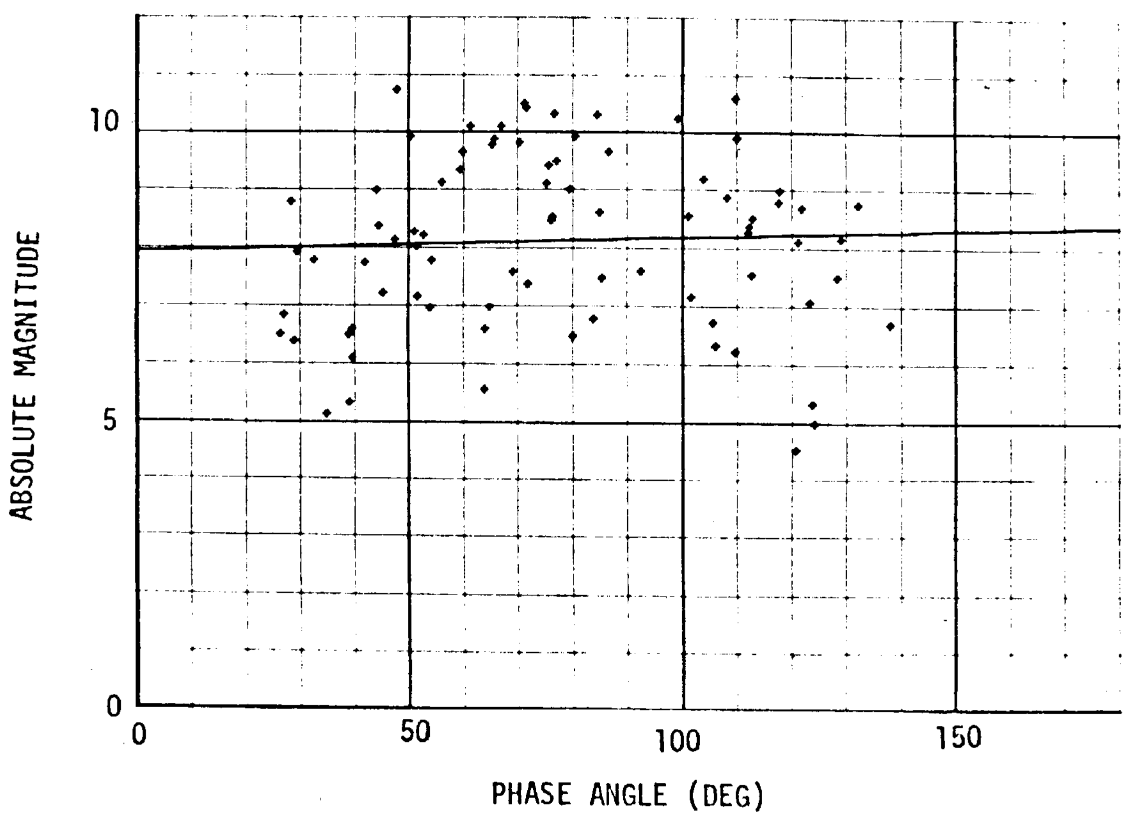

Figure 10. Maximum Absolute Magnitude Versus Phase Angle for Satellite #613 (σ_{yx} = 1.51 Mag.)

82 Observations

alternate in brightness. Sometimes flashes are observed off of the ends. If one is interested in estimating the size of a rocket body the minimum absolute magnitude is the best indicator.

Most objects will show an occasional flash from flat or curved specular surfaces but some objects are frequent or continual flashers. Figure 11 shows the observations of the minimum absolute magnitude of 2403, another Agena. It can be seen that there are a sequence of observations which appear about 2.5 magnitudes brighter than is usual. At the times of anomalous brightness the object shows a variation of several magnitudes compared to its usually steady appearance. Figures 12 and 13 show the minimum and maximum magnitudes for Alouette 1 (424) an oblate spheroid with minimum and maximum cross-sectional areas of 0.7 and 0.9 m^2. This satellite is covered with solar cells so that it almost always flashes. The flashing causes the large scatter in the minimum absolute magnitudes. Since the eye underestimates the brightness of a short flash, the real variation in brightness is undoubtedly even greater than shown here.

Many rocket bodies do not tumble fast enough for the observer to unambiguously observe the maximum and minimum brightness during each tumble. Figures 14 and 15 show the minimum and maximum absolute magnitudes as a function of date for a rocket whose spin rate suddenly increased from zero somewhere between MJD 37600 and 37900. After the spin-up the reported minima decreased and the reported maxima increased. This is just what one would expect since the spin-up allowed the extremes to be observed during the pass.

Figure 16 shows the minimum absolute magnitude at a phase angle of 90^0 for all catalog objects which have been well observed. The absolute

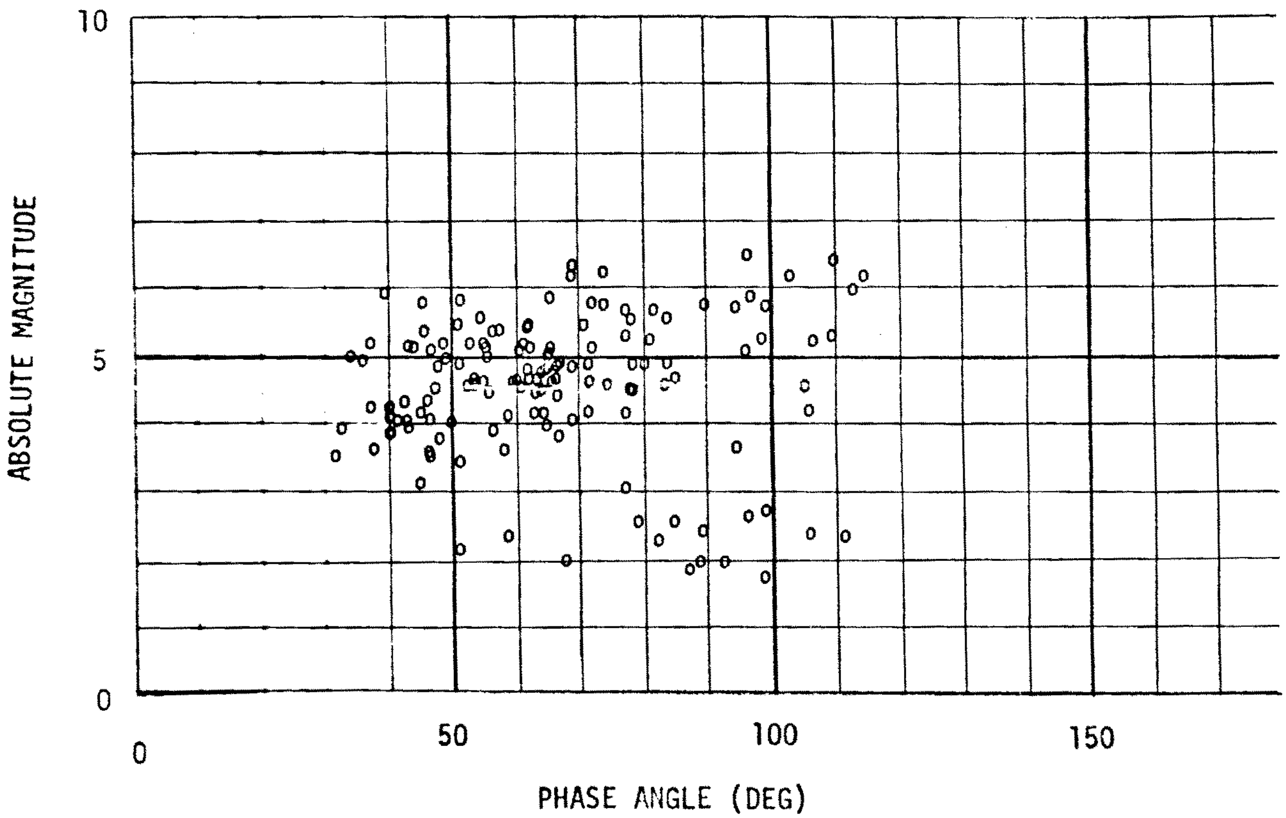

Figure 11. Minimum Absolute Magnitude Versus Phase Angle for Satellite #2403 (σ_{yx} = 1.0 Mag.)
142 Observations

Note Sequence of Flashes 2.5 Magnitudes Brighter Than Usual

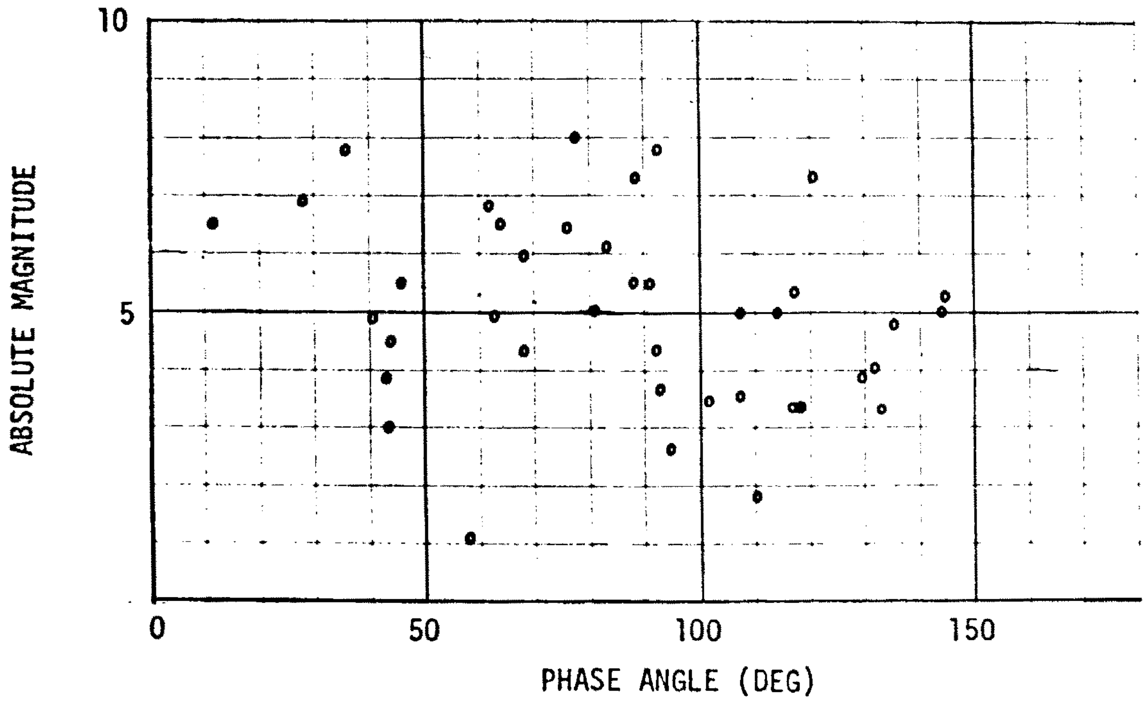

Figure 12. Minimum Absolute Magnitude Versus Phase Angle for Satellite #424 (σ_{yx} = 1.58 Mag.)

41 Observations

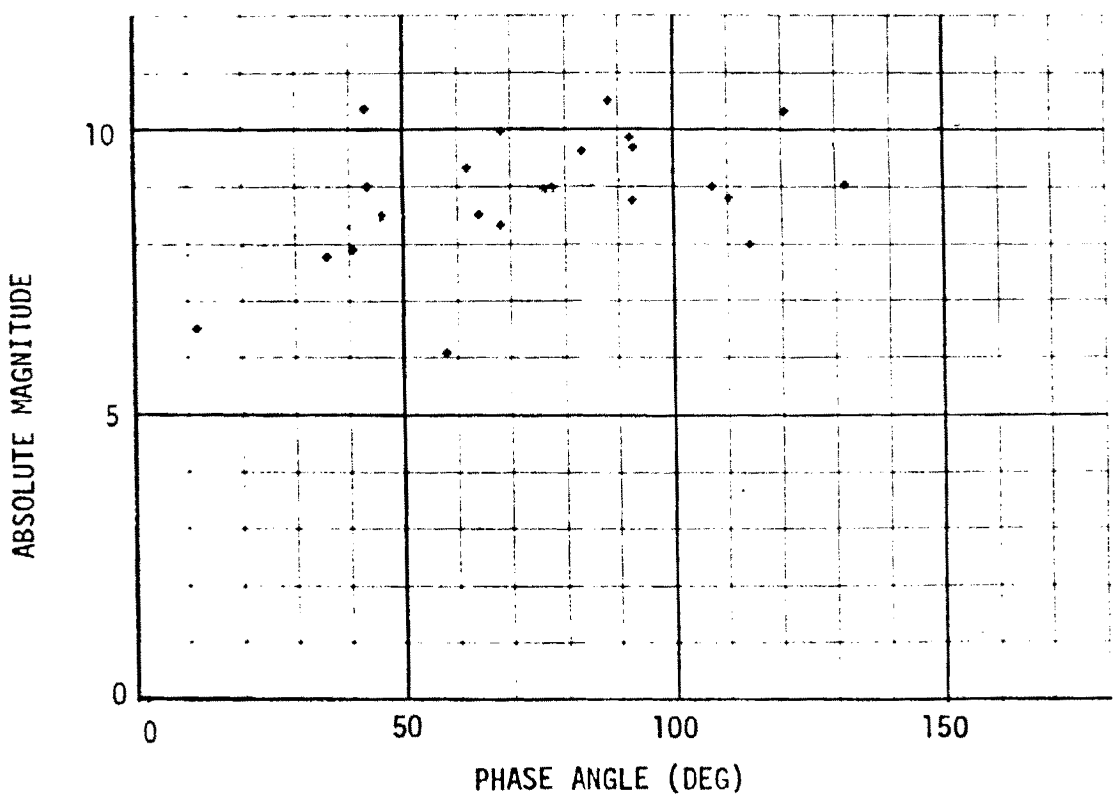

Figure 13. Maximum Absolute Magnitude Versus Phase Angle for Satellite #424 (σ_{yx} = 1.00 Mag.)
23 Observations

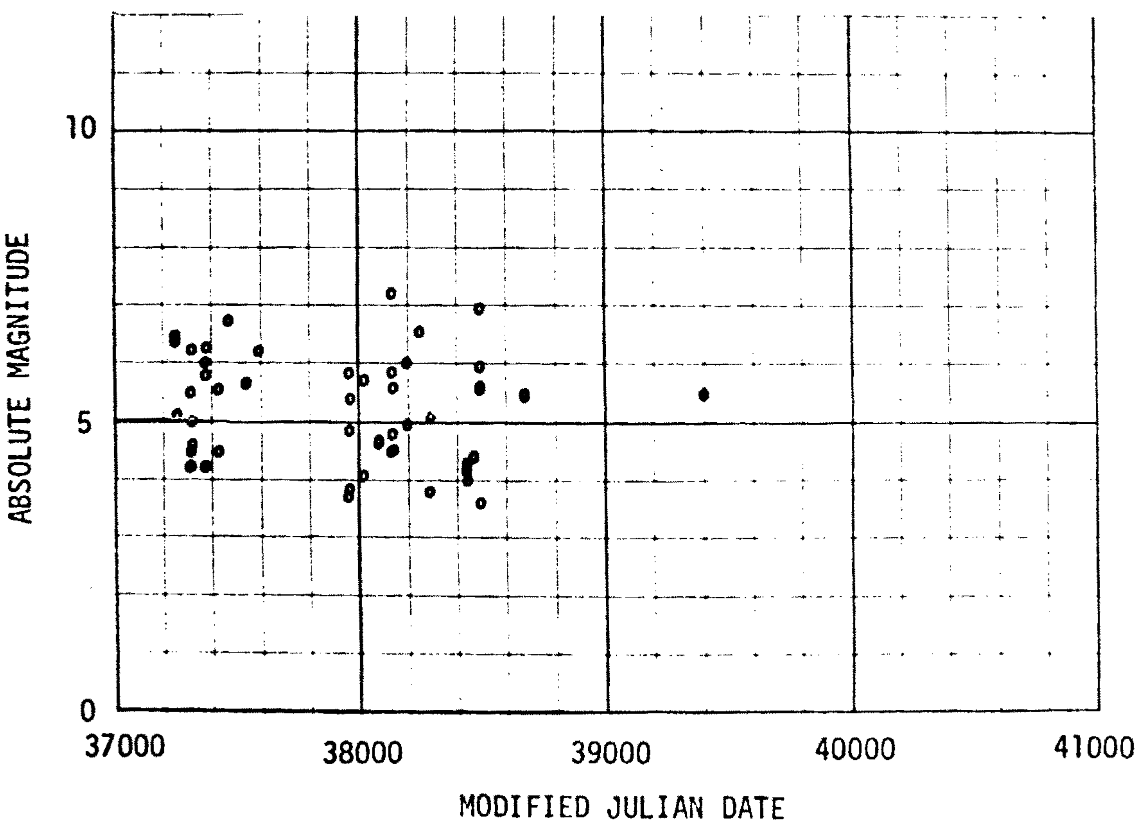

Figure 14. Minimum Absolute Magnitude Versus Date for Satellite #59

Spin Up Occurred Between MJD 37600 and 37900

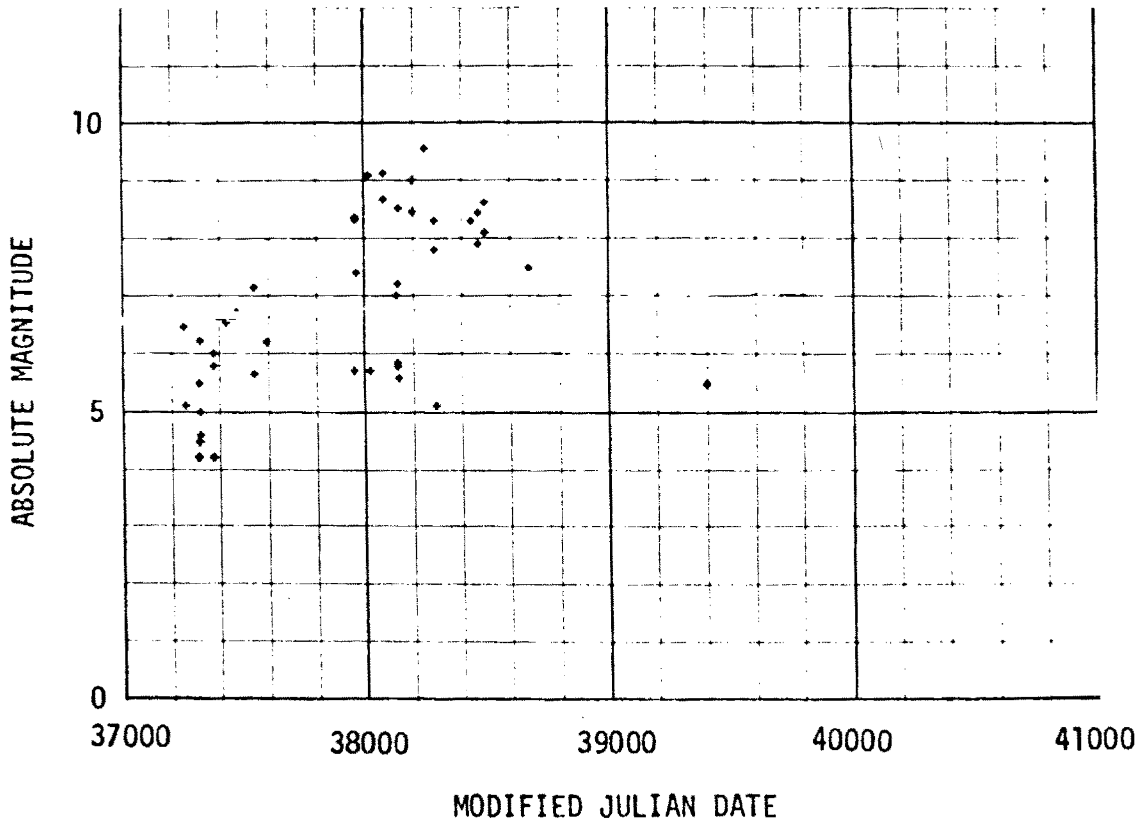

Figure 15. Maximum Absolute Magnitude Versus Date for Satellite #59

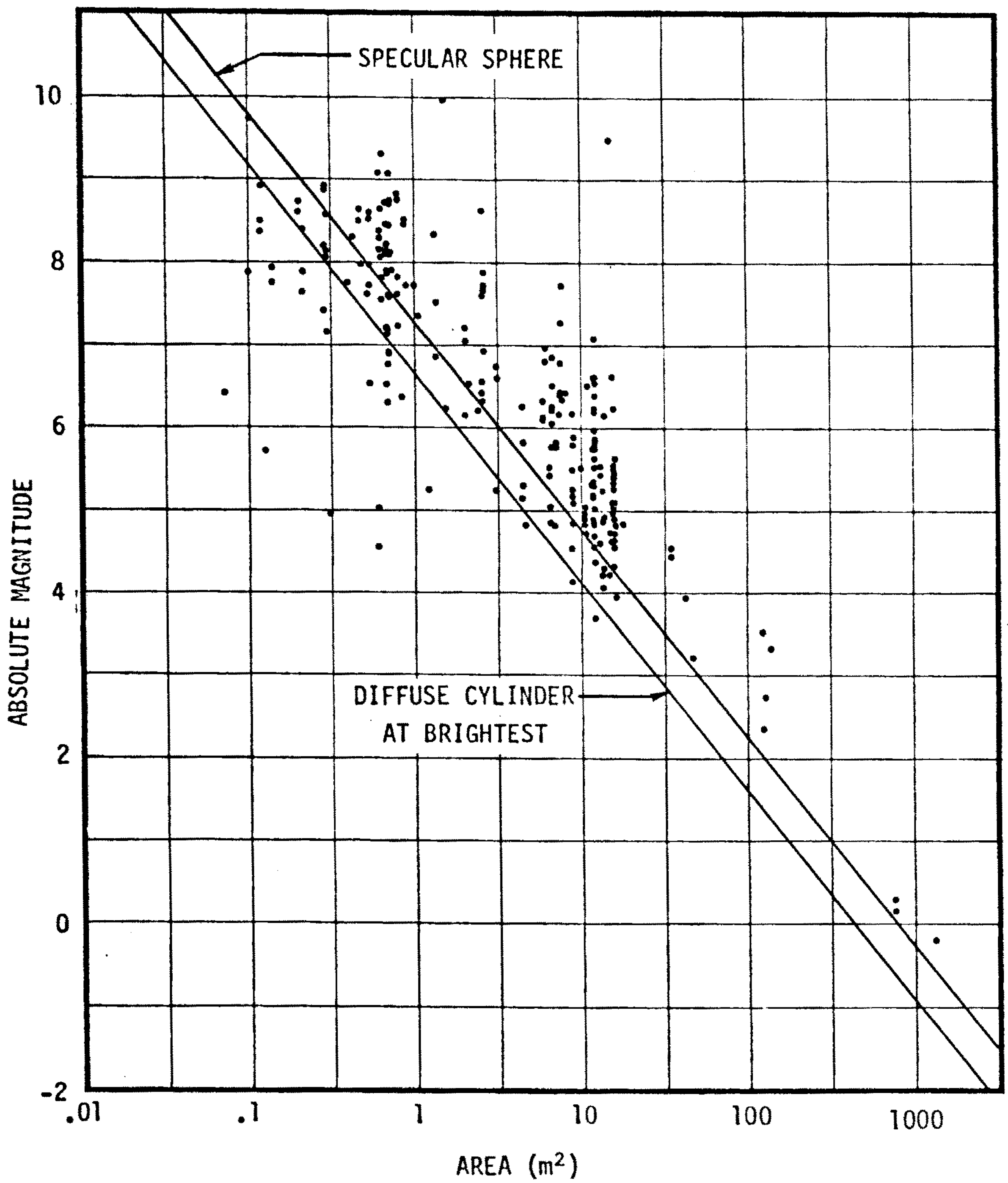

Figure 16 Area vs Absolute Minimum Magnitude for Typical Satellite Population Used In Study

magnitudes are plotted against their areas. The lines are drawn for the specular sphere and the diffuse cylinder at brightest, both with $\gamma=1$. Scatter about the line is caused by the different shapes and reflectivities. Points which fall far below the lower line belong to flashing objects.

5. CONCLUSION

The results of this study indicate that the satellite observations under consideration are remarkably consistent. However, there was strong evidence of observer bias when reporting the observed magnitude of the same satellite. In general, the observations tended to be of very high quality, thus indicating the ability of volunteer observers to contribute valuable scientific data. For instance, when the observations reported by all observers were aggregated, it was possible to distinguish between nearly identical objects. The observational results were also compared with theoretical satellite brightness predicted by equations derived by the authors. Consistent agreement with theoretical calculations was found.

REFERENCES

1. Williams, J. G., McCue, G. A. 1966, Planet. Space Sci. 14, 839
2. McCue, G. A., Williams, J. G., Duprie, H. J., Hoy, R. C. 1965, NAA Rept. No. SID 65-1176
3. Williams, J. G., McCue, G. A., Morford, J. M. 1968, MAR Rept. SD 68-629
4. Pilkington, J. A. 1964, Planet. Space Sci. 12, 597
5. ______1965, 13, 541
6. ______1966, 14, 1281
7. ______1967, 15, 1535
8. Meeus, J. 1966, Ciel et Terre, 95
9. ______1968, Ciel et Terre, 1
10. Pees, J. M., King-Hele, D. G. 1963, Planet. Space Sci. 11, 1053
11. King-Hele, O. G., Quinn, E. 1964, Planet. Space Sci. 12, 681
12. ______1965, Planet. Space Sci. 13, 707
13. ______1966, Planet. Space Sci. 14, 817
14. Quinn, E., King-Hele, D. G. 1967, Planet. Space Sci. 15, 1181
15. McCue, G. A., Williams, J. G., Hoy, R. C. 1965, Sky and Telescope 30, 88
16. Tousey, R. 1957, J. Opt. Sac. An, 47, 261
17. Russell, H. '1. 1916, Ap. J. 43, 173
18. Giese, R. H. 1963, S.A.O. Special Report No. 127
19. Liemohn, B. 1968, Icarus 9, 217
20. Williams, J. G,., 1969 , Proceedings of Universitv of Miami Symposium on Optical Properties of Orbiting Satellites, D. Duke and K. Kissell Eds. (Aerospace Research Laboratory 69-0078) 31
21. Emmons, R., Rogers, C. L., Preski, R. J. 1967, AJ 72, 939
22. Dixon, W. J., ed. 1964, BMD Biomedical Computer Programs, School of Medicine, Univ. of Calif. at L. A.
23. Dixon, W. J., Massey, Jr., F. J. 1968, Introduction to Statistical Analysis, McGraw-Hill, New York